INTEGRAL METHOD OF COMPUTING THE DIFFUSION OF A POLYMER IMPURITY IN THE TURBULENT FLOW AT A PERMEABLE SURFACE

V. A. Ioselevich and S. L. Mashkov

UDC 536.526:532.72

An integral method of computing turbulent flow parameters at a permeable wall through which fluid with a drag-reducing polymer impurity is delivered is proposed on the basis of an asymptotic analysis of velocity and concentration profiles. Formulas are obtained to compute the effective impurity concentration.

One of the rational methods of inserting an impurity into a near-wall flow is the distributed delivery of a solution through a permeable (porous or perforated) wall surface. In application to the utilization of active impurities reducing turbulent friction (particularly polymer admixtures), this method is still of interest because blowing the fluid carrying the impurity through the permeable wall also results in drag diminution.

Up to now there have been no sufficiently well-founded schemes for computing such flows, which did not permit reliable prediction of the possible reduction in friction by polymer admixtures. An integral method is proposed below for computation of the turbulent boundary layer that relies on an asymptotic analysis of the velocity and concentration profiles of the active impurity at the permeable wall. For simplicity just the normal injection of a dilute polymer solution is examined and it is assumed that the injection velocity v_w is small compared with the free stream velocity u_δ. This permits conservation of the boundary layer approximation.

It is known [1] that dimensionless transverse mixing path length distributions $l^+ = l u_\tau/\nu$, tangential stress $\tau^+ = \tau/\tau_w$ and diffusion impurity flow $q^+ = q/q_w$ are sufficient for computation of the velocity and concentration profiles as well as for determining the integral characteristics of the flow (such as $\delta^+ = \delta u_\tau/\nu$, $\omega = u_\tau/u_\delta$, $c_f = 2\omega^2$, etc.). Indeed from

$$\tau = \nu\left(1 + \frac{\nu_t}{\nu}\right)\frac{\partial u}{\partial y}, \quad q = -\nu\left(\frac{1}{\mathrm{Sc}} + \frac{1}{\mathrm{Sc}_t}\,\frac{\nu_t}{\nu}\right)\frac{\partial c}{\partial y}, \quad \nu_t = l^2\frac{\partial u}{\partial y}$$

there follows that the solutions of the equations

$$\frac{du^+}{dy^+} = \frac{2\tau^+}{1 + \sqrt{1 + 4l^{+2}\tau^+}}, \quad u^+ = \frac{u}{u_\tau}, \quad y^+ = \frac{y u_\tau}{\nu}, \tag{1}$$

$$\frac{dc^+}{dy^+} = q^+\left(\frac{1}{\mathrm{Sc}} + \frac{1}{\mathrm{Sc}_t}\,l^{+2}\frac{du^+}{dy^+}\right)^{-1}, \quad c^+ = \frac{(c_w - c)\,u_\tau}{q_w} \tag{2}$$

with boundary conditions $u^+ = 0$ and $c^+ = 0$ at the wall (for $y^+ = 0$) yield profiles of u^+ and c^+ in transverse sections of the flow.

Following [2], we assume that the distributions of τ^+ and q^+ in a gradient-free boundary layer on a permeable surface has the form

$$\begin{gathered} \tau^+ = \tau^+(\xi,\ u^+,\ v_w^+) = 1 - 3\xi^2 + 2\xi^3 + v_w^+ u^+ (1 - \xi)^2, \\ q^+ = q^+(\xi,\ c^+,\ v_w^+) = 1 - 3\xi^2 + 2\xi^3 + v_w^+ c^+ (1 - \xi)^2, \\ \xi = \frac{y}{\delta}, \quad v_w^+ = \frac{v_w}{u_\tau}. \end{gathered} \tag{3}$$

Strictly speaking, (3) are valid only under the condition of equality of the thicknesses of the dynamic and diffusion boundary layers. In connection with the fact that propagation of the relatively slow-moving polymer impurity (the coefficient of its molecular diffusion

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M. V. Lomonosov Moscow State University Institute of Mechanics. Translated from Inzhenerno-fizicheskii Zhurnal, Vol. 60, No. 2, pp. 186-190, February, 1991. Original article submitted August 1, 1989.

in water and other low-molecular solvents is ordinarily of the order of 10^{-7}-10^{-8} cm²/sec, i.e., Sc $\gg$ 1) beyond the dynamic layer limits can be neglected, it is sufficient that the whole wall surface flowed around by the stream, starting from its leading edge, be perforated for the mentioned condition to be satisfied.

To describe the transverse transfer of momentum we take (see [3]) a mixing path distribution in the form

$$l^+ = \delta^+ l_1(\xi)[1 - \exp(-y^+ F\sqrt{1 + v_w^+ u^+}/A)], \tag{4}$$

where $\ell_1(\xi) = a\,\mathrm{th}\,(\kappa\xi/a)$, $\xi = y/\delta$, $\kappa \approx 0.4$, $a \approx 0.075$, $A \approx 26$, and the parameter F takes account of the impurity "activity" ($F < 1$ for polymer admixtures reducing friction while $F = 1$ for a passive impurity). Formulas relating F to the stream parameters, the molecular characteristics of the dissolved polymer, and its effective concentration c_e are presented in [3].

It follows from an asymptotic analysis of the solutions of (1) and (2) by using (3) and (4), performed analogously to [1], that in the outer boundary layer domain

$$u^+ = \alpha + \frac{v_w^+ \alpha^2}{4} + f(\xi,\ v_w^+),$$

$$\alpha = \frac{1}{\varkappa}\ln y^+ + B(F,\ v_w^+), \tag{5}$$

$$\frac{c}{c_w} = 1 - \left(\frac{v_w^+\alpha + 2}{v_w^+\beta + 2}\right)^2,\quad \beta = \frac{1}{\varkappa}\ln \delta^+ + B(F,\ v_w^+). \tag{6}$$

Exactly as in [1], explicit formulas can be obtained for the parameter $B(F,\ v_w^+)$ and the function $f(\xi,\ v_w^+)$ in the dimensionless profiles (5) and (6). The results of computations performed by using these formulas permit proposal, in particular, of the simple approximation

$$B(F,\ v_w^+) \approx A_1(v_w^+)\,F^{-m(v_w^+)} - C_1(v_w^+), \tag{7}$$

in which

$$A_1(v_w^+) \approx 3{,}31\exp(-3{,}88 v_w^+) - \exp(-125 v_w^+) + 6{,}5,$$

$$C_1(v_w^+) \approx 2{,}4\exp(-4{,}18 v_w^+) - 1{,}3\exp(-92 v_w^+) + 2{,}5,$$

$$m(v_w^+) \approx \frac{31{,}85 v_w^+ + 0{,}6}{63{,}7 v_w^+ + 1}.$$

The correction function $f(\xi,\ v_w^+)$ in (5) is independent, in practice, of the dimensionless injection velocity $v_w^+ = v_w/u_\tau$. This means that exactly as the polymer admixtures, weak normal injection is an "interior" factor exerting substantial influence on the flow structure only near the wall. Consequently

$$f(\xi,\ v_w^+) \approx f(\xi,\ 0) = f_0(\xi) = \int_0^\xi \left[\frac{\sqrt{\tau_0^+(\xi)}}{l_1(\xi)} - \frac{1}{\varkappa\xi}\right] d\xi. \tag{8}$$

Here $\tau_0^+(\xi) = \tau^+(\xi,\ u^+,\ 0)$ is the dimensionless tangential stress distribution in the boundary layer on an impermeable surface.

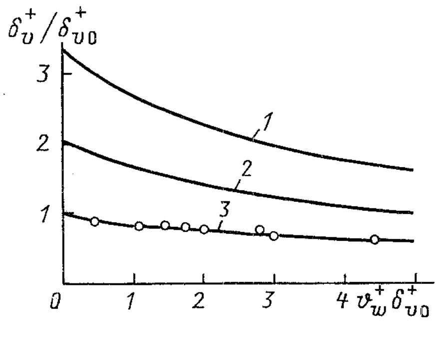

Fig. 1. Dependence of the relative viscous sublayer thickness δ_v^+/δ_{v0}^+ on a permeable plate on a dimensionless injection $v_w^+\delta_{v0}^+$ for different values of the parameter F. 1) F = 0.3; 2) 0.5; 3) 1.0. The points are experimental data from [4].

It must be emphasized that the asymptotic profile of the relative concentration (6) is obtained by using the condition $Sc \gg 1$. It describes well the concentration distribution beyond the limits of the diffusion sublayer even for $Sc \gtrsim 100$. The polymer admixtures ordinarily applied to reduce turbulent friction are characterized by the values $Sc \sim 10^5$-10^6 and sometimes even higher.

In connection with the profile (6), the question occurs of the effective polymer substance concentration c_e in a transverse boundary layer section. The parameter F in the mixing path length (4) depends on precisely this concentration (see [3]) and therefore, the quantity $B(F, v_w^+)$ in (5) and (6) and in the long run, the reduction of turbulent friction does too. Polymer admixtures interact with turbulent perturbations within the limits of the near-wall layer, and more accurately, in the buffer domain of the viscous sublayer and its adjoining domain of the logarithmic velocity distribution. The greatest intensity of "work" of the macromolecules occurs in the neighborhood of the viscous sublayer boundary. Consequently, it can be considered that the polymer substance concentration on this boundary is effective, i.e., $c_e = c(\delta_v)$.

It is easy to show that the equality

$$F\delta_v^+\left\{v_w^+\left[\frac{1}{\varkappa}\ln\delta_v^+ + B(F,\ v_w^+)\right] + 2\right\} = 2\delta_{v0}^+, \tag{9}$$

in which $\delta_{v0}^+ \approx 56$ is the dimensionless viscous sublayer thickness on an impermeable surface for $F = 1$ is valid for the dimensionless viscous sublayer thickness. Taking account of (9) an equation to determine the effective admixture concentration follows from (6)

$$\frac{c_e}{c_w} = 1 - \left[\frac{2\delta_{v0}^+}{F\delta_v^+(v_w^+\beta + 2)}\right]^2. \tag{10}$$

Dependences computed by means of (7) and (9) for the relative viscous sublayer thickness δ_v^+/δ_{v0}^+ on a permeable plate on the parameter $v_w^+\delta_{v0}^+$ are presented in Fig. 1 for different values of F. Shown there for comparison are test data [4] corresponding to distributed injection of a fluid with a passive impurity. It is seen that for $F = 1$ utilization of asymptotic formulas results in good agreement with experiment. In the presence of an active impurity the dimensionless viscous sublayer thickness increases while injection results in its contraction.

Using the velocity profile (5) and the expression (8), the dimensionless displacement thickness $R^* = \delta^* u_\delta/\nu$ and loss of momentum $R^{**} = \delta^{**}u_\delta/\nu$ can be represented in the form

$$R^* = \delta^+ G_1, \quad R^{**} = \delta^+(G_1 - \omega G_2), \tag{11}$$

where

$$G_1 = G_{10} + \frac{v_w^+}{2}\left(\frac{\beta}{\varkappa} - \frac{1}{\varkappa^2}\right);$$

$$G_2 = G_{20} + v_w^+(E_{10}\beta + E_{20}) + \frac{v_w^{+2}}{2}\left(\frac{\beta^2}{\varkappa^2} - \frac{3\beta}{\varkappa^3} + \frac{3}{\varkappa^4}\right);$$

$$G_{10} = \frac{1}{\varkappa} + f_0(1) - \int_0^1 f_0(\xi)\,d\xi \approx 3{,}47;$$

$$G_{20} = \frac{2}{\varkappa^2} + \frac{2}{\varkappa}f_0(1) + f_0^2(1) - 2f_0(1)\int_0^1 f_0(\xi)\,d\xi + \int_0^1 f_0^2(\xi)\,d\xi + \frac{2}{\varkappa}\int_0^1 f_0(\xi)\ln\xi\,d\xi \approx 23{,}46;$$

$$E_{10} = \frac{2}{\varkappa^2} + \frac{1}{\varkappa}f_0(1) + \frac{1}{\varkappa}\int_0^1 f_0(\xi)\ln\xi\,d\xi \approx 17{,}04;$$

$$E_{20} = -\frac{3}{\varkappa^3} - \frac{1}{\varkappa^2}f_0(1) + \frac{1}{2\varkappa^2}\int_0^1 f_0(\xi)\ln^2\xi\,d\xi \approx -60{,}81.$$

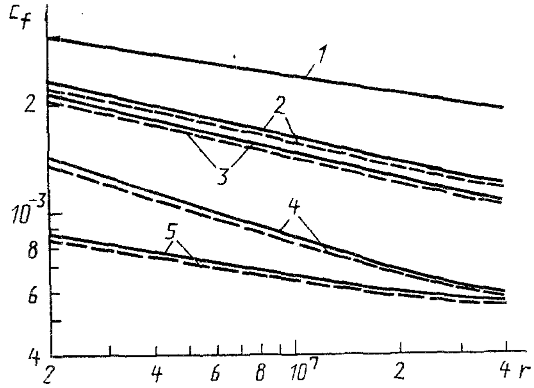

Fig. 2. Distributions of the local friction coefficient $c_f(r)$ on a permeable plate for a distributed passive impurity delivery and a dilute solution of polyethylene oxide ($u_\delta = 10^3$ cm/sec; $v_w/u_\delta = 10^{-3}$; 1) $v_w = 0$; 2) passive impurity injection; 3) distributed delivery of polymer impurity, polyethylene oxide with mean molecular mass $M = 2 \cdot 10^6$ g/mole for $c_w = 3 \cdot 10^{-6}$ g/cm³; 4) $3 \cdot 10^{-5}$ g/cm³, and 5) $3 \cdot 10^{-4}$ g/cm³). Solid lines are computation by the integral method, and dashes are numerical solution of the boundary layer equations.

Entering into the system of integral relations governing the distributions of δ^+, ω, and c_e/c_w in the turbulent boundary layer on a permeable plate are:

the momentum equation

$$\frac{dR^{**}}{dr} = \omega^2 + \frac{v_w}{u_\delta}, \quad r = \frac{xu_\delta}{\nu}, \tag{12}$$

the drag law following from (5) and (8)

$$4\omega\,[\beta + f_0(1)] = 4 - v_w^+\beta^2, \tag{13}$$

Expressions (7)-(11) and formulas for the parameter F (see [3]). The quantity R^{**} in some (initial) section of the boundary layer as well as values of u_δ, v_w and c_w should be given for integration of the system mentioned.

In order to confirm the assumptions made in the proposed computational scheme (in particular, the assumptions about τ^+, q^+ and the effective concentration c_e), a numerical solution of the complete system of boundary layer equations is performed by using the expression (4) for the mixing path. Diagrams of the drag $c_f(r)$ obtained by an integral method and from the numerical solution are displayed in Fig. 2. The discrepancies between corresponding c_f distributions do not exceed 3-4% in a sufficiently broad range of r (from $2 \cdot 10^6$ to $4 \cdot 10^7$), which indicates the acceptability of the assumptions made. The integral method yields a barely higher value of the local friction drag coefficient c_f everywhere.

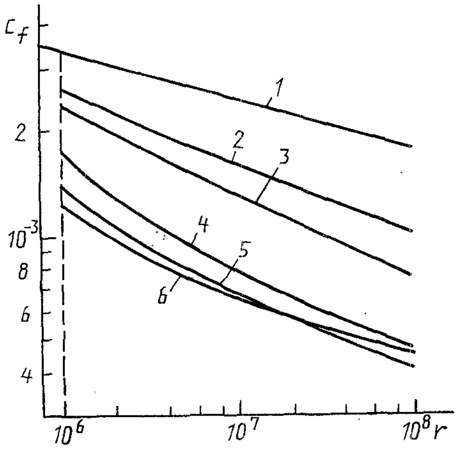

Fig. 3. Diagrams of the drag $c_f(r)$ of a permeable plate for different concentrations of a polyethylene oxide solution delivered through the permeable plate surface ($u_\delta = 10^3$ cm/sec; $v_w/u_\delta = 10^{-3}$; 1) $v_w = 0$; 2) passive impurity injection; 3) $c_w = 6 \cdot 10^{-6}$ g/cm³; 4) $3 \cdot 10^{-5}$ g/cm³; 5) $6 \cdot 10^{-5}$ g/cm³; 6) 10^{-4} g/cm³).

Drag diagrams computed by an integral method are shown in Fig. 3 for a permeable plate whose boundary layer is turbulized for $r = 10^6$. It is seen that as the concentration c_w of the delivered solution increases in each boundary layer section, the effect of reducing the friction first grows and then when the quantity c_e exceeds the optimal macromolecule concentration, starts to be reduced. The joint influence of injection and polymer admixtures permits obtaining a substantial reduction in turbulent friction even for very large values of v_w/u_δ.

Notation. u_δ is the free stream velocity; u_τ is the dynamic velocity; τ_w is the tangential stress on the wall; q_w is the diffusion impurity flux through the permeable wall; v and v_t are molecular and turbulent viscosity coefficients; c_w is the impurity concentration on the wall; δ and δ_v are boundary layer and viscous sublayer thicknesses; Sc and Sc_t are the molecular and turbulent Schmidt numbers; z and y are the longitudinal and transverse coordinates and c_f is the local friction drag coefficient.

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MATHEMATICAL MODELLING OF NONISOTHERMAL TURBULENT ONE- AND TWO-PHASE SWIRLING FLOWS

V. V. Novomlinskii

UDC 532.529

A mathematical model is developed and realized numerically for turbulent gas-dispersed nonisothermal swirling flows on the basis of Navier—Stokes type equations by using a modified k—ε turbulence model. Corrections taking account of the influence of particles and the flow swirling on k, ε are introduced into these latter. A finite-difference method of controlled volume is used to solve the equations. Computations are compared with experimental data on swirling single-phase flows in a cylindrical channel. Data are obtained about the influence of the nonisothermy on the length of the recirculation zone.

Introduction. Two-phase (gas—solid or liquid particles) turbulent high-temperature swirling flows are utilized extensively in plasma technology, plasma chemistry, and powder metallurgy. The investigation of such flows and the subsequent development of existing technologies on this basis can be carried out, in particular, by constructing numerical models of these processes and executing numerical experiments by using such models, as would permit determination of the most important flow characteristics, the velocity, phase temperatures, powder concentration, etc.

Different turbulent two-phase jet flows, computation methods, and results of numerical modelling are presented in [1]. The extensively known Prandtl mixing-path model modified by G. N. Abramovich for gas-dispersed flows, is used as the closure model.

Turbulent swirling flows are utilized for intensification of heat and mass transfer processes in different apparatus as well as for electric arc stabilization in plasmatrons. It is known that strongly swirling flows are characterized by the occurrence of recirculation

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Ordzhonikidze Moscow Aviation Institute. Translated from Inzhenerno-fizicheskii Zhurnal, Vol. 60, No. 2, pp. 191-197, February, 1991. Original article submitted December 9, 1989.